CFA LEARNING OUTCOMES DECODED

In our series *Learning Outcomes Decoded* we break down a single Learning Outcome Statement (LOS) from the CFA level I curriculum. This article is written by M. Emrul Hasan, CFA, FRM, PhD. Emrul is the Content Director of the CFA team at the Princeton Review. He is a professor of economics and finance at the University of British Columbia, Canada.

QUANTITATIVE METHODS: PROBABILITY CONCEPTS

LOS: Calculate and interpret an updated probability using Bayes' formula

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Almost every mock exam of Level I CFA exam has questions around Bayes' formula. The formula provides a way to revise existing probability given new or additional information. In finance the formula is frequently used to evaluate the risk of borrowing, assess the risk of default, and help managerial decision making. Recently, the formula has become a useful component of machine learning.

Bayes' Formula

To understand Bayes' formula properly, we need to understand the concepts of *Prior probability* and *Posterior probability*.

- *Prior probability* is the probability of an event occurring before we get any new information. It is the best assumed probability of a particular outcome (e.g., probability that stock market will be up) from an event based on current knowledge before more information is gathered.

- *Posterior probability* is the *updated* probability of a particular outcome from an event after considering new information. This probability is calculated by updating the prior probability using the Bayes' formula below (e.g., probability that stock market will be up "given that" the economy is in recession).

Bayes' formula gives the "updated" probability of an outcome based on new information that may be related to that event or on some hypothetical new information assuming that the information is true.

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Formula of Bayes’ theorem and example:

$$P(A|B) = \frac{P(A) \times P(B|A)}{P(B)}$$

Let’s assume we have received a report that the probability that the stock market will be up next year is 40% (= $P(A)$). We also found that, in years when we see stock market is up, the probability of a recession is 15% (= $P(B|A)$). Next, we gathered some new information about the economy that there is a 55% probability the economy will be in recession next year (= $P(B)$). Now, given this new information, the updated probability that the stock market will be up next year can be calculated as

$$P(A|B) = \frac{P(A) \times P(B|A)}{P(B)} = \frac{0.40 \times 0.15}{0.55} = 0.11$$

Given the new information that the economy will be in recession next year, the “updated” probability that the stock market will be up next year is only 11%, lower than the previously assumed probability of 40%.

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PRACTICE QUESTION

A drilling company has estimated a 40% chance of being successful (striking oil) for their new well. A detailed test has been scheduled for more information. Historically, 60% of successful wells have had detailed tests, and 20% of unsuccessful wells have had detailed tests. Given that this well has been scheduled for a detailed test, what is the *most likely* updated probability that the well will be successful?

A. 40.00%

B. 66.70%

C. 24.00%

B is correct.

$$P(Successful|DetailedTest) = \frac{P(Successful) \times P(DetailedTest|Successful)}{P(DetailedTest)} = \frac{0.40 \times 0.60}{?} = ?$$

We are missing the unconditional probability of a detailed test, but we can apply the *total probability rule* that we learn in the same module which is

$$P(DetailedTest) = P(Successful) \times$$
$$P(DetailedTest|Successful) + P(Unsuccessful) \times P(DetailedTest|Unsuccessful) =$$
$$0.40 \times 0.60 + 0.60 \times 0.2 = 0.36$$

So,

$$P\left(Successful|DetailedTest\right)=\frac{P\left(Successful\right)\times P\left(DetailedTest|Successful\right)}{P\left(DetailedTest\right)}=\frac{0.40\times 0.60}{0.36}=0.667$$

The revised probability of success (from the original estimate of 40%), given that this well has been scheduled for a detailed test, is 66.70%.